CBSE ACADEMY PLUS

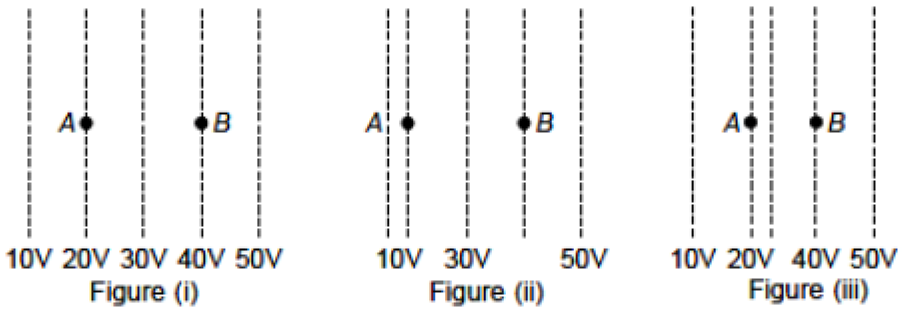
UNIT 02

Electrostatic Potential and Capacitance

SECTION – A

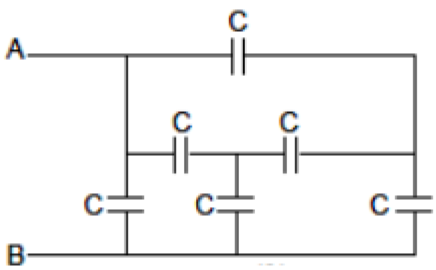
Questions 1 to 10 carry 1 mark each.

1. Figures show some equipotential lines distributed in space. A charged object is moved from point A to point B.



(a) The work done in Fig. (i) is the greatest.
(b) The work done in Fig. (ii) is least.
(c) The work done is the same in Fig. (i), Fig.(ii) and Fig. (iii).
(d) The work done in Fig. (iii) is greater than Fig. (ii) but equal to that in Fig. (i).
Ans: (c) The work done is the same in Fig. (i), Fig.(ii) and Fig. (iii).
The work done by the electric field on the charge will be negative.
$W_{electrical} = -\Delta U = -q\Delta V = q(V_{initial} - V_{final})$ Here initial and final potentials are same in all three cases and the same charge is moved, so work done is same in all three cases.

2. Find the equivalent capacitance of the system across the terminals A and B. All the capacitors have equal capacitances.

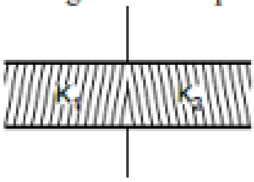


(a) 2 C (b) 4 C (c) 3 C (d) 5 C
Ans: (a) 2 C

3. The capacitors of capacitance 4 F, 6 F and 12 F are connected first in series and then in parallel. What is the ratio of equivalent capacitance in the two cases?
(a) 2 : 3 (b) 11 : 1 (c) 1 : 11 (d) 1 : 3
Ans: (c) 1 : 11

$C_s = 2$ F, $C_p = 22$ F, $\frac{C_s}{C_p} = 1 : 11$

4. A parallel plate capacitor with air as medium between the plates has a capacitance of 10 μF. The area of capacitor is divided into two equal halves and filled with two media having dielectric constant $k_1 = 2$ and $k_2 = 4$ as shown in the figure. The capacitance of the system will now be



(a) 10 μF (b) 20 μF (c) 30 μF (d) 40 μF

Ans: (c) 30 μF

5. A point P lies at a distance x from the mid point of an electric dipole on its axis. The electric potential at point P is proportional to

(a) $\frac{1}{x^2}$ (b) $\frac{1}{x^3}$ (c) $\frac{1}{x}$ (d) $\frac{1}{x^{1/2}}$

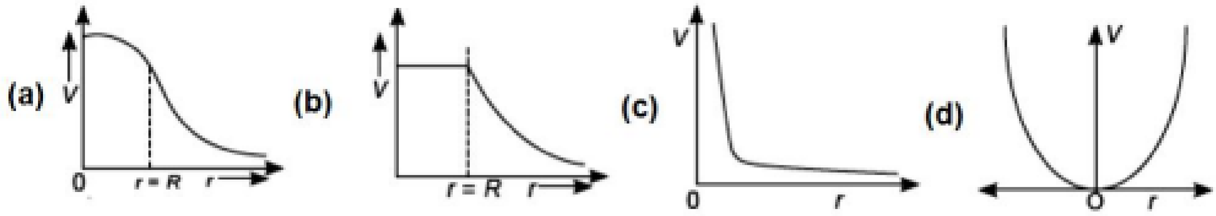
Ans: (a) $\frac{1}{x^2}$

6. A charge Q is located at the centre of a circle of radius r.
The work done in moving a test charge q_0 from point A to point B (at opposite ends of diameter AB) so as to complete a semicircle is $\left[k = \frac{1}{4\pi\varepsilon_0}\right]$

(a) $k\frac{q_0 Q}{r}$ (b) $\frac{Qq_0}{r^2}$ (c) kq_0Qr (d) Zero

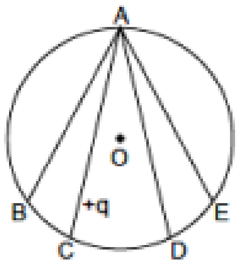
Ans: (d) Zero

7. Each of the graphs below represents the variation of electrostatic potential with distance in the region around a source charge, that is, either a point charge or a continuous charge distribution. Identify the most relevant $V(r)$ vs r graph due to a uniform charged insulated sphere.



Ans: (a)

8. In the electric field of a point charge q , a certain charge is carried from point A to B, C, D and E. Then the work done



(a) is least along the path AB.
(b) is least along the path AD.
(c) is zero along all the paths AB, AC, AD and AE.
(d) is least along AE.

Ans: (c) is zero along all the paths AB, AC, AD and AE.
ABCDE is an equipotential surface and on equipotential surface no work is done in shifting a charge from one place to another.

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

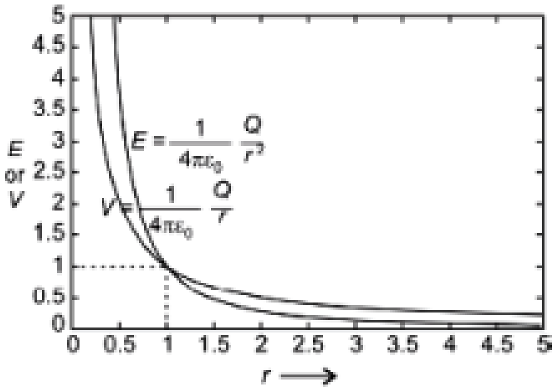
9. **Assertion (A):** Work done in moving a charge around a closed path, in an electric field is always zero.
Reason (R): Electrostatic force is a conservative force.
Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

10. **Assertion (A):** A parallel plate capacitor is connected across a battery. A dielectric slab of dielectric constant K is introduced between the plates. The energy stored becomes 1/K times.
Reason (R): Energy does not depend on dielectric constant.
Ans: (d) Assertion (A) is false but reason (R) is true.

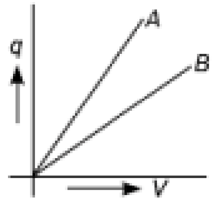
SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Draw a plot showing the variation of (*i*) electric field (E and (*ii*) electric potential (V) with distance r due to a point charge Q.
Ans:



12. The given graph shows that variation of charge q versus potential difference V for two capacitors C_1 and C_2. The two capacitors have same plate separation but the plate area of C_2 is double than that of C_1. Which of the lines in the graph correspond to C_1 and C_2 and why?

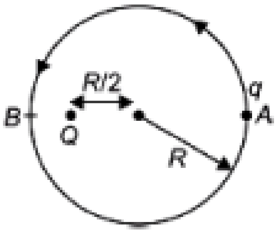


Ans: Since, $C \propto A$ [Area]
$\therefore C_2 = 2C_1$ $(\because A_2 = 2A_1)$

but $C = \frac{q}{V}$ So, more slope represents more capacitance.

Hence, A represents C_2 and B represents C_1.

13. There is a point charge Q at a distance $\frac{R}{2}$ from the centre of a circle of radius R. Another point charge q is to be moved from A to B, where A and B are two points on the circle diametrically opposite to each other. How
much work is done by the electrostatic force exerted by Q on q?



Ans:
Work done, $W = -\frac{q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$, here, $r_2 = \frac{R}{2}$ and $r_1 = \frac{R}{2} + R = \frac{3R}{2}$

$\therefore \quad W = -\frac{Qq}{4\pi\varepsilon_0}\left[\frac{2}{3R} - \frac{2}{R}\right] = \frac{Qq}{3\pi\varepsilon_0 R}$

14. A charge Q is given to three capacitors C_1, C_2 and C_3 connected in parallel. Determine the charge on each.

Ans: In parallel combination of capacitors, the total charge $Q = Q_1 + Q_2 + Q_3$.
As potential difference across each capacitor is same, therefore
$\because C_{eq} = C_1 + C_2 + C_3$

$$\therefore \quad P.D. = V = \frac{Q}{C_1 + C_2 + C_3} = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3}$$

$\therefore$ Charge on C_1, $\quad Q_1 = \frac{C_1}{C_1 + C_2 + C_3} Q$

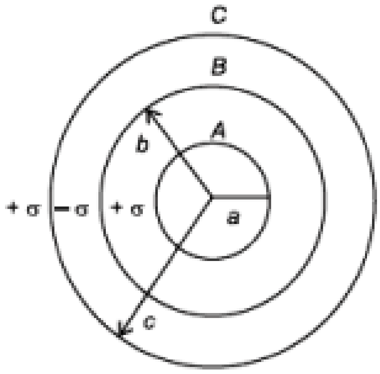
Charge on C_2, $\quad Q_2 = \frac{C_2}{C_1 + C_2 + C_3} Q$

Charge on C_3, $\quad Q_3 = \frac{C_3}{C_1 + C_2 + C_3} Q$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Three concentric metallic shells A, B and C of radii a, b and c ($a < b < c$) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively as shown in the figure.
If shells A and C are at the same potential, then obtain the relation between the radii a, b, c.



Ans: Given that, A, B and C are three concentric shells of radii a, b and c. σ, $-\sigma$ and σ are the charge densities on them respectively.
Also $V_C = V_A$ (given)

Now $V_A = \frac{\sigma a}{\varepsilon_0} - \frac{\sigma b}{\varepsilon_0} + \frac{\sigma c}{\varepsilon_0}$ $\left(\because V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{\sigma.4\pi r^2}{4\pi\varepsilon_0 r} = \frac{\sigma r}{\varepsilon_0}\right)$

(As A is the inner most shell and potentials of B and C will be added up with that of A.)

Also, $V_C = \frac{\sigma c}{\varepsilon_0} - \frac{\sigma b^2}{\varepsilon_0 c} + \frac{\sigma a^2}{\varepsilon_0 c}$

(As for both A and B shells, C lie at outside points.)

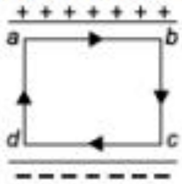
As $V_A = V_C$, we have,

$$\frac{\sigma a}{\varepsilon_0} - \frac{\sigma b}{\varepsilon_0} + \frac{\sigma c}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0} + \sigma\left(\frac{a^2 - b^2}{\varepsilon_0 c}\right) \Rightarrow \frac{\sigma}{\varepsilon_0}(a-b) = \frac{\sigma}{\varepsilon_0}\left[\frac{(a+b)(a-b)}{c}\right]$$

$\therefore$ $c = a + b$

16. (a) Obtain the expression for the energy stored per unit volume in a charged parallel plate capacitor.

(b) The electric field inside a parallel plate capacitor is E. Find the amount of work done in moving a charge q over a closed rectangular loop $a\ b\ c\ d\ a$.



Ans: (a) Work done by a source of potential V in storing an additional charge dq

$dW = Vdq$

$$dW = \frac{q}{C}dq \qquad \left(\because V = \frac{q}{C}\right)$$

Total work done in storing a charge Q to an uncharged capacitor is

$$W = \int_0^Q \frac{q}{C}dq = \frac{1}{2}\frac{Q^2}{C}$$

This work done gets stored in the form of electrostatic energy in the capacitor.

Hence $U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$

As $C = \frac{\varepsilon_0 A}{d}$ and $V = Ed$

$\therefore$ Energy density $(u) = \frac{\text{Energy stored}}{\text{Volume}}$

$$u = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}\left(\frac{A\varepsilon_0}{d}\right)(Ed)^2}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

where u is the energy density.

(b) Electric field is conservative. So, the total amount of work done in moving a charge q over a closed loop abcda will be zero.

$W_{loop} = W_{ab} + W_{bc} + W_{cd} + W_{da}$

$W_{loop} = 0 + q\,(\Delta V_{bc}) + 0 + q\,(\Delta V_{da})$

As $\Delta V_{bc} = -\Delta V_{da}$, 2nd and 4th terms in RHS of above equation will cancel out.

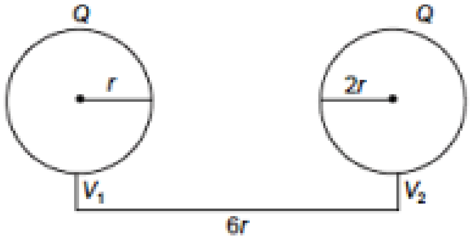
$\therefore W_{loop} = 0$

17. Two metal spheres A and B of radius r and $2r$, whose centres are separated by a distance of $6r$, are given charge Q each and are at potential V_1 and V_2. Find the ratio of V_1/V_2. These spheres are connected to each other with the help of a connecting wire keeping the separation unchanged, what is the amount of charge that will flow through the wire?

Ans:

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{2}{1}$$

$$\because \quad V_1 > V_2$$



Since after connecting these two spheres, the potential of each sphere will be equal, i.e.

$$V_1 = V_2$$

$$\therefore \quad \frac{kQ'_1}{r} = \frac{kQ'_2}{2r}$$

$$\therefore \quad Q'_1 = \frac{Q'_2}{2}$$

Electric charges are conserved.

$$\because \quad Q + Q = Q'_1 + Q'_2$$

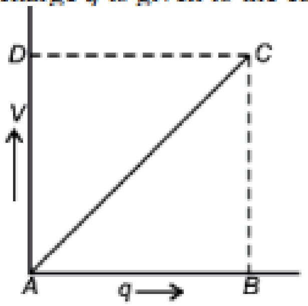
$$2Q = \frac{3Q'_2}{2} \Rightarrow Q'_2 = \frac{4}{3}Q$$

Charge flowing $= Q'_2 - Q = \frac{4Q}{3} - Q = \frac{Q}{3}$

OR

Show by graph how q given to a capacitor varies with its potential difference.
Using the graph or otherwise, prove that the energy of a capacitor is 1/2 CV^2. Calculate the energy density of the electrostatic field in a parallel plate capacitor.

Ans: As $q = CV$, V versus q will be a straight line. During the process of giving charge q to the capacitor, the potential difference across the capacitor rises linearly from 0 to V. So, the charge q is given to the capacitor at on average potential difference $V/2$.



$[\because q = CV]$

From the graph, areaof $ABC = \frac{1}{2}qV$

$\therefore$ Energy stored $= q\left(\frac{V}{2}\right) = \frac{1}{2}CV^2$

$$\text{Energy density, } u = \frac{\text{Energy}}{\text{Volume}} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{1}{2}\frac{\varepsilon_0 A}{d}\frac{(Ed)^2}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

SECTION – D

Questions 18 carry 5 marks.

18. Derive the expression for the energy stored in a parallel plate capacitor of capacitance C with air as medium between its plates having charges Q and $\frac{1}{2}\varepsilon_0 E^2 Ad$ where A is the area of each plate and d is the separation between the plates.
How will the energy stored in a fully charged capacitor change when the separation between the

plates is doubled and a dielectric medium of dielectric constant 4 is introduced between the plates?

Ans:

Suppose at any instant of time potential difference between the capacitor plates be V. Then the amount of work required to supply a charge dq to the capacitor is given by

$dW = Vdq$

To supply a charge Q, the work done is given by

$$W = \int_0^Q V dq = \int_0^Q \frac{q}{C} dq \qquad \left(\because V = \frac{q}{C}\right)$$

$$W = \frac{1}{2}\frac{Q^2}{C} \qquad (\because Q = CV)$$

This work done gets stored in the form of electrostatic potential energy

or $$W = \frac{1}{2}CV^2$$

$\therefore$ $$U = \frac{1}{2}KC_0V^2 \qquad (\because C = KC_0)$$

As $$C_0 = \frac{\varepsilon_0 A}{d},\ V = Ed$$

$$U = \frac{1}{2}K\frac{\varepsilon_0}{d}E^2d^2A = \frac{1}{2}K\varepsilon_0E^2Ad$$

As we know $$C_0 = \frac{\varepsilon_0 A}{d}$$

When the separation is doubled, then

$$C'_0 = \frac{\varepsilon_0 A}{2d} = \frac{C_0}{2}$$

On introducing the dielectric medium of dielectric constant $K = 4$ between the plates,

$$C = KC'_0$$

$$E'_v = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{KC_0'} \qquad \left[\because C'_0 = \frac{C_0}{2}\right]$$

$$E'_v = \frac{1}{2}\left(\frac{Q^2}{4\frac{C_0}{2}}\right) = \frac{1}{2}\left(\frac{1}{2}\frac{Q^2}{C_0}\right) \qquad [\because K = 4]$$

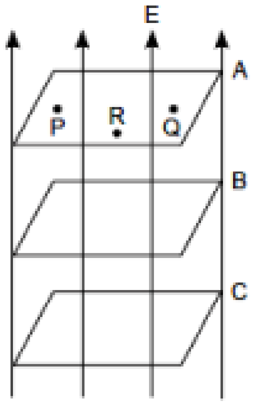
$$E'_v = \frac{1}{2}E_n$$

The energy stored in the capacitor reduces to one-half of its original value.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Consider three surfaces A, B and C are placed perpendicular to the electric field 'E'. These surfaces are at equal distance from each other.



(i) If a charge $+q$ is moved from 'P' to point 'Q' via a point 'R' on the surface A then how much work is done in this case?

(a) qE (b) qV (c) qF (d) Zero

(ii) What will happen if the electric field become parallel to these surfaces in (i)?

(a) Work done will be zero. (b) Force due to electric field will be zero.
(c) Some work will be done (d) None of the above

(iii) Consider these surfaces are not at equal distances, then what will it represent?

(a) Electric field is zero. (b) Electric field is constant.
(c) Electric field is not constant. (d) Potential difference between P and Q is positive.

(iv) What will happen if surfaces 'B' and 'C' intersect in the same field?

(a) There will be two directions of electric fields which is not possible in equipotential surfaces.
(b) There will be two directions of electric fields, indicating equipotential surfaces.
(c) It will not affect direction of electric field.
(d) None of the above.

OR

(iv)Nature of equipotential surface for a point charge is

(a) ellipsoid with charge at foci.
(b) sphere with charge at the centre of the sphere.
(c) sphere with charge on the surface of the sphere.
(d) plane with charge on the surface.

Ans : (i) (d) As surface 'A' is perpendicular to electric field therefore this is an equipotential surface. Work done to move a charge on equipotential surface is zero.

(ii) (c) In this case the electric field will exert a force on charge and the work will be done on it.

(iii)(c) If the surfaces are not equidistant that means the electric field is not constant.

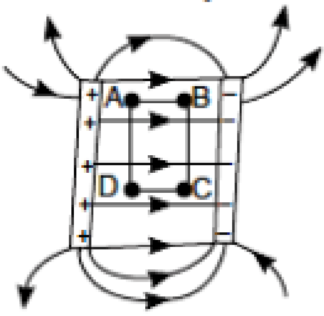
(iv)(a) If surfaces B and C intersect then these will have two directions of electric field which is not possible.

OR

(iv) (b)

20. Electric field between oppositely charged parallel conducting plates: When two plane parallel conducting plates, having the size and spacing as shown in the figure, are given equal and opposite charges, the field between and around them is approximately as shown, while most of the charge accumulates at the opposite faces of the plates and the field is essentially uniform in the space between them, there is a small quantity of charge on the outer surfaces of the plates and a certain spreading or "fringing of the field at the edges of the plates.

As the plates are made larger and the distance between them diminished, the fringing becomes relatively less. This kind of arrangement is called capacitor.



Now if two plates are separated by a distance '3d', and are maintained at a potential difference 'V', then answer the following questions.

(i) Capacitance of a parallel plate capacitor depends on

(a) electric charge stored.

(b) electric field between the plates.
(c) potential difference applied.
(d) electrical permittivity of the medium between the plates.

(ii) If two protons are placed at points A and B respectively, then which one will experience more force?
(a) Proton at point A
(b) Proton at point B
(c) Proton at points A and B both experiences same force.
(d) None of the above

(iii) When both the protons are released then which one will gain more K.E. just before striking the negative (–ve) plate?
(a) Proton released from point A
(b) Proton released from point B
(c) Proton released from points A and B will gain equal kinetic energy.
(d) No one will gain kinetic energy.

(iv) If one proton is moved from (I) A to B, (II) B to C, and (III) along ABCD, then work done in cases I, II and III respectively are
(a) eED, Zero –eED
(b) eED, Zero, Zero
(c) –eED, +eED, Zero
(d) eED, –eED, Zero

OR

(iv) Which property of electric field is shown by part (iii) and part (iv)
(a) Non-conservative nature
(b) Conservative nature
(c) Dissipative nature
(d) None of the above

Ans: (i) (d)
(ii) (c) Both protons will experience same force.
Reason: $F = qE$; E = constant; $q = +e$ (same)
(iii)(a) Reason: $\because V_D = V_A > V_B = V_C$
[In the direction of electric field potential decreases]
$V_A > V_B$ or $(P.D)_{AP2} > (P.D)_{BP2}$
$\therefore$ Gain in K.E. $= q \times$ P.D.
$\therefore$ Gain in K.E. of proton released from point A will be more.

(iv)(b) Reason: $W_{A\to B} = e(V_B - V_A)$ $\begin{bmatrix} \because W = q \times P.D. \\ E = \dfrac{V}{d} \end{bmatrix}$

$= e[E.2d - E.d] = eE.d$
$\because V_B = V_C$,
$\therefore W_{BC} = 0$
$W_{ABCD} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$
$= eE.d + 0 - eE.d + 0 = 0$

OR

(iv) (b)